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EFFECT OF MATRIX CRACKS ON STIFFNESS DEGRADATION OF LAMINATED COMPOSITE BEAMS

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Keywords: Laminated beams, Matrix cracks, Multiscale approach, Damage mechanics

ABSTRACT

The structural response of laminated composite beams with off-axis matrix cracks is analyzed in this study. For that, a damage mechanics-based multiscale approach using a 2D finite-element-based cross-section analysis is developed. By using this approach, it is shown how the thickness and orientation of cracked plies and the cross-section geometry affect the degradation level of the beam stiffness properties. Beams with thin-walled box and symmetric airfoil cross-sections made of symmetric multidirectional laminates are analyzed in this study in order to get a better understanding of the structural response of damaged wind turbine blades.

1 INTRODUCTION

Laminated composite beams are commonly used in a variety of engineering applications, such as the load carrying girder in wind turbine blades. These structures are made of composite laminates consisting of multiple plies oriented in different directions and bonded together through a matrix. When these laminates are subjected to quasi-static or fatigue loading conditions, matrix cracks at the off-axis plies are one of the first damage mechanisms that develop [1], see stage I in Fig. 1. These matrix cracks can lead to a decrease of the material stiffness and trigger the developing of other damage mechanisms such as delaminations [2] and fibre breakage [3], which in turn contribute to the final failure of the material [1], see stages II and III in Fig. 1. Hence, modelling of matrix cracks is necessary for understanding and predicting the structural behavior of composite laminated structures.

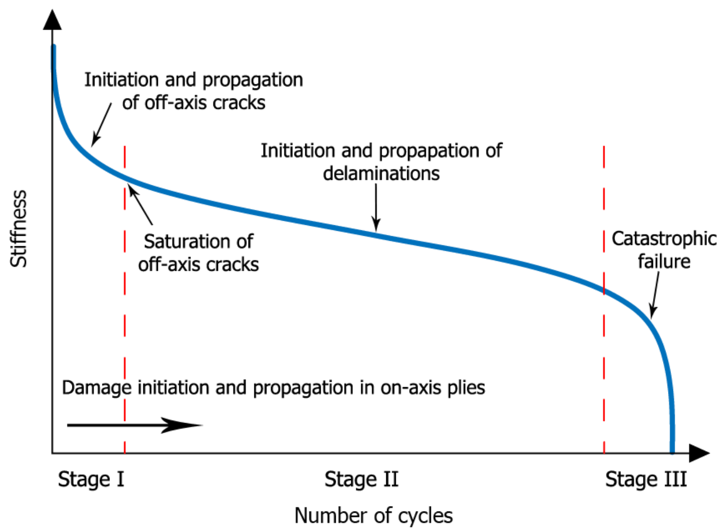

Figure 1: Damage evolution and stiffness degradation of multidirectional composite laminates. Figure taken from [4].

From a material point of view, the effects of off-axis matrix cracks on the mechanical properties of multidirectional laminates have been analyzed experimentally over many years, e.g., [5-9]. It has been shown, for example, that the off-axis cracks initiate during the first number of cycles causing a degradation of the material properties and evolves rapidly until a crack saturation point is reached, from which the mechanical response of the laminate tends to a stable condition, see stage I in Fig. 1. Different analytical and numerical models have been proposed in order to predict this behavior, e.g., [10-16]. Carraro and Quaresimin [16], for example, proposed an analytical model based on a bi-dimensional optimal shear lag analysis (as defined in [17]), which allows predicting the material property degradation of any multidirectional symmetric laminate with off-axis matrix cracks.

From a structural point of view, the stiffness redistribution in laminated composite beams due to matrix cracking has been studied considering prismatic rectangular cross-section beams [18], prismatic thin-walled beams with box and airfoil cross-sections [19], and more complex beams such as wind turbine blades [20]. These studies showed how the beam cross-section stiffness properties decrease with the density of the matrix cracks [18-20] and how the level of degradation depends on other parameters, such as the ply angle and the percentage of cracked plies [19]. However, little attention has been given to analyzing the effect of other factors, such as the laminate thickness and the contribution of each laminate within the cross-sections, on the beam stiffness degradation.

This paper presents a study about the effects of the thickness and orientation of the cracked plies and the cross-section geometry on the stiffness degradation of different matrix-cracked composite beams. For that, a damage mechanics-based multiscale approach using a 2D finite-element-based cross-section analysis is developed. This study aims to gain a better insight into the structural response of damaged wind turbine blades.

2 METHODS

A damage mechanics-based multiscale approach is developed to quantify the structural response of laminated composite beams with off-axis matrix cracks, see Fig. 2. This approach is used to analyze the effects of the cross-section geometry and the orientation and thickness of the matrix-cracked plies on the stiffness degradation of the beams.

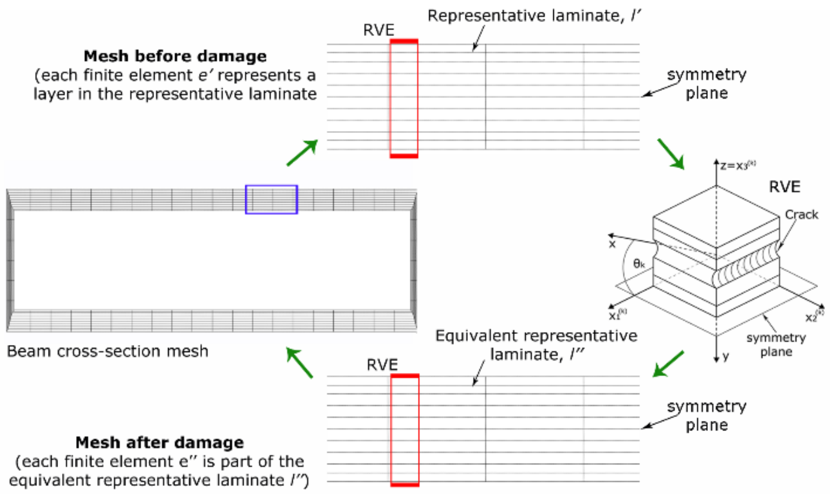

Figure 2: Damage mechanics-based multiscale model for laminated composite beams by using a 2D finite-element-based cross-section analysis.

In the proposed approach, the beam stiffness properties are obtained by using BECAS (BEam Cross-section Analysis Software) tool [21], which is a general-purpose cross-section analysis tool developed at DTU that estimates efficiently and accurately the mechanical response of complex beam-like structures. During the modeling, each cross-section is discretized in finite elements, which are assumed to form representative laminates (RL), see Fig. 2. Each of these RLs are further discretized in representative volume elements (RVE) containing a crack spacing l_k. The degraded compliance matrix of each of the RLs due to the damage in the RVEs is calculated by applying the damage mechanics model developed by Carraro and Quaresimin in [16], which predicts the stiffness degradation of symmetric multidirectional laminates with off-axis matrix cracks. Subsequently, the effects of the damaged RLs on the structural response of the cross-sections (and therefore of the beam) are sequentially coupled by using BECAS.

The proposed model is applied on beams with thin-walled box and symmetric airfoil cross-sections, which are selected to describe in a simpler way the complexity of the geometry and material distribution of wind turbine blades, see Fig. 3. The symmetric airfoil cross-section was defined based the FFA-W3-211 airfoil developed by the Aeronautical Research Institute of Sweden [22]. For that, the pressure part of this airfoil is deleted and the suction part is copied and flip vertically to form the symmetric airfoil. Two shear webs located at 17% and 50% of the chord length respectively are also included in this cross-section, see Fig. 3. Moreover, two thin-walled box cross-sections are also analyzed. The first one is a thin-walled semi-rectangular box (TWSRB) cross-section formed by the webs and caps of the symmetric airfoil cross-section, see Fig. 3; whereas, the second one is a thin-walled rectangular box (TWRB) cross-section formed by the webs of the symmetric airfoil cross-section but assuming straight caps, see Fig. 3. For all cases, the non-dimensional coordinates are scaled using a chord length of 0.59m. This chord length was selected to obtain realistic dimensions of the TWRB beam, which will be manufactured to validate this model in future studies.

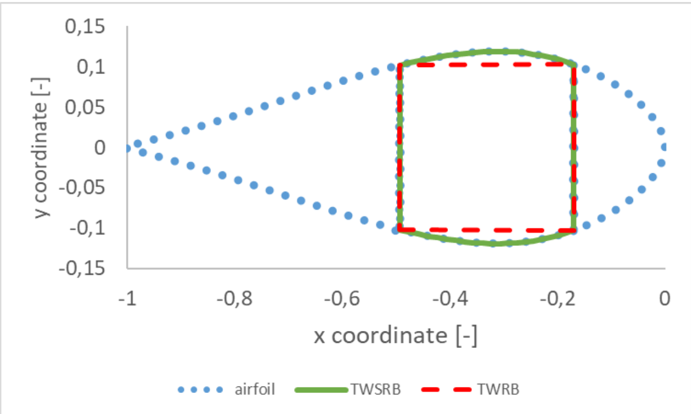

Figure 3: Analyzed cross-section geometries: symmetric airfoil with shear webs (airfoil), thin-walled semi-rectangular box (TWSRB), and thin-walled rectangular box (TWRB).

A base laminate with $[0/\theta/0/-\theta]_s$ lay-up is used in this study, where θ is the ply angle. This laminate is chosen to study the effects of both the ply angle and the laminate thickness on the cross-section response (see Sections 3.1 and 3.2, respectively) while keeping the symmetric condition of the laminate required by the applied damage mechanics model [16]. The laminate is assumed to be made of a unidirectional glass-epoxy material [23], whose mechanical properties are shown in Table 1.

E_1(GPa)	E_2(GPa)	G_{12}(GPa)	υ_{12}(-)	t_p(mm)*
48.830	14.070	5.200	0.308	0.445

Table 1: Mechanical properties of the used glass-epoxy material [23]. *Assumed value.

Moreover, it is assumed that all laminates within the cross-section have the same layup and thickness. This is far from a realistic wind turbine blade design, where the caps are thick laminates with most fibres in the longitudinal direction and webs and airfoil parts usually are sandwich panels with triax and/or biax layers. Nevertheless, this assumption allows to analyse in a simple way the effect of the cross-section regions and the cross-section geometry on the cross-section stiffness degradation, see Section 3.3. Furthermore, it is also assumed that the matrix cracks were located only in the off-axis plies (i.e., no matrix crack in 0° plies) and that all the cracked laminates have the same crack density. This last assumption is indeed not realistic because the damage might develop differently in the laminate depending on the internal forces, internal defects, environmental conditions, among other factors, which induce variability in the fatigue response of the material [24]. However, this assumption allows analysing what would be the maximum effect of this type of damage on the structural response of the cross-section.

3 RESULTS AND DISCUSSION

The effect of the matrix crack density ρ_k on the cross-section stiffness properties (i.e. bending stiffness in *x* direction, k_{44}, bending stiffness in *y* direction, k_{55}, and torsional stiffness, k_{66}) of several composite laminated beams was analysed, see Fig. 4-8.

For all evaluated cases, it was found that these properties decrease as ρ_k increases until reaching a crack density value from which the properties do not change substantially any more (see Fig. 1), which agrees with what was found in [18-20]. This crack density value can be related to the maximum number of cracks that can initiate in each off-axis ply within the beam laminates. As explained at the material length scale in [25], the stiffness of a multidirectional laminate is constant before damage starts and starts reducing where multiple matrix cracking takes place. Later, the stiffness becomes constant again after reaching a crack saturation condition, which depends on the layup, ply thickness and materials used in the laminate. This behavior keeps in this way until other damage mechanisms, such as delaminations, develop and start contributing as well to the stiffness degradation of the laminate [1].

On the other hand, it was also observed that the degradation level of the beam properties (i.e., $k_{ii}/k_{0,ii}$, where $i = 4, 5$ or 6, k is the actual stiffness after cracking and k_0 is the initial stiffness) due to the matrix cracks can vary according to the configuration of the beam laminates (e.g., layup, ply thickness) and the cross-section geometry. A more detailed analysis of these factors in the structural response of the beams is presented below.

3.1 Ply angle effect

Wind turbine blades are normally made of multidirectional laminates with plies orientated to 0° and ±45°. However, new designs are exploring the possibility of using plies oriented in other directions in order to improve the aeroelastic and structural response of these structures. For this reason, the effect of the cracked-ply angle θ on the stiffness degradation of blade-like composite beams caused by matrix cracks is analysed in this section. In this analysis, it is assumed that all laminates within the cross-section are the same and have the same off-axis crack density.

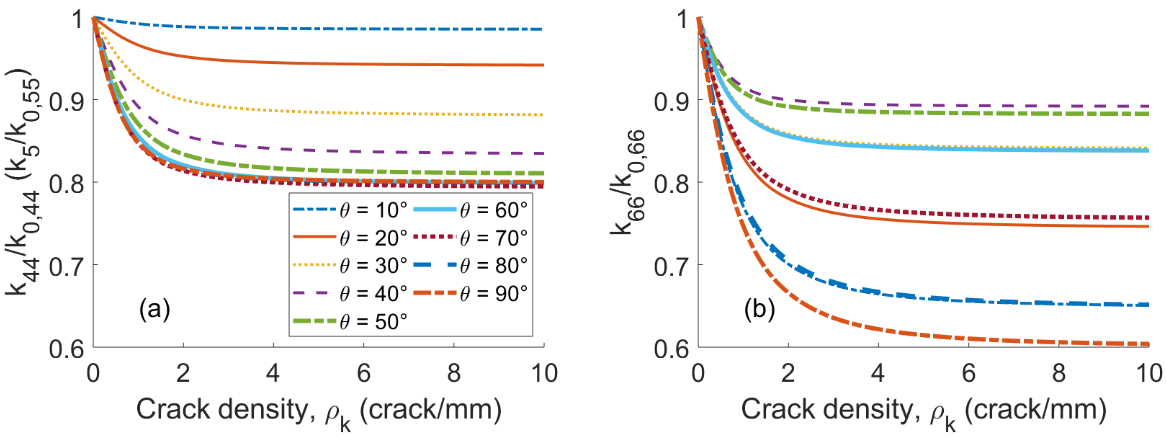

Figure 4: Effect of matrix crack density and ply angle θ on the cross-section (a) bending stiffness, k_{44} and k_{55}, and (b) torsional stiffness, k_{66}, of laminated beams made of $[0/\theta/0/-\theta]_s$ laminates.

The degradation level of the bending stiffness, $k_{44}/k_{0,44}$ and $k_{55}/k_{0,55}$, of beams made of $[0/\theta/0/-\theta]_s$ laminates as a function of the matrix crack density ρ_k is shown in Fig. 4-a for different ply angle θ. As shown in this figure, $k_{44}/k_{0,44}$ and $k_{55}/k_{0,55}$ decrease as the cracked-ply angle θ increases for any ρ_k value. This is because the fibers in the cracked plies contribute less in the extension and bending stiffness of the material when θ increases. In addition, it is also observed that $k_{44}/k_{0,44}$ and $k_{55}/k_{0,55}$ tend to the same value for high values of θ. For this group of laminates, for example, the highest degradation of the bending stiffness is 20%, meaning that 80% of the bending stiffness remains. Note that $k_{44}/k_{0,44}$ and $k_{55}/k_{0,55}$ are the same in this case because the beam is made of the same type of laminate and all laminates within it are assumed to degrade in the same way.

Moreover, the degradation level of the torsion stiffness, $k_{66}/k_{0,66}$, is shown in Fig. 4-b. As shown in this figure, $k_{66}/k_{0,66}$ decreases as the cracked-ply angle θ moves away to the right and to the left of 45° (i.e., for $\theta < 45°$ and $\theta > 45°$), being the lowest value at $\theta = 90°$. This is because there is a lower contribution of the fibers within the cracked plies in the main shear direction of the material when θ is different from 45°, which leads to a reduction of the shear stiffness of the material. For this group of laminates, the highest degradation of the beam torsion stiffness is around 40%.

3.2 Laminate thickness effect

In wind turbine blades, several laminates with different layups and thicknesses are normally used. In this section, the effects of scaling the size of the initial laminate on the stiffness degradation of blade-like composite beams due to matrix cracks are analysed. For that, two types of scaling methods are studied, in which the fiber orientation and volume fraction of the initial lamina are kept constant. The first method is the *ply level scaling*, where the thickness of each ply within the initial laminate is multiplied by means of a factor p (e.g., $[0_p/\theta_p/0_p/-\theta_p]_s$). Whereas the second method is the *sub-laminate scaling*, where the thickness of the plies within the initial laminate stays the same but the number of initial laminates (i.e., sub-laminates) is multiplied by means of a factor q (e.g., $[0/\theta/0/-\theta]_{s,q}$). In this analysis, it is also assumed that all laminates within the cross-section are the same and have the same off-axis crack density.

The effect of the *ply level scaling* on the stiffness property degradation of laminated beams due to matrix cracks is shown in Fig. 5. As shown in this figure, the thinner the plies within the beam laminates, the lower the degradation of the cross-section stiffness properties, especially at low ρ_k values. This agrees with what was observed numerically and analytically at the material length-scale in [26], where it was found that the degradation level of the material increases with the increase of the cracked-ply thickness. Moreover, Fig. 5 also shows that the degradation level of the beam properties decreases as ρ_k increases and tends to the same value for all thicknesses. This means that the maximum crack density of the laminate may also depends on the thickness of the cracked plies, as discussed in [25].

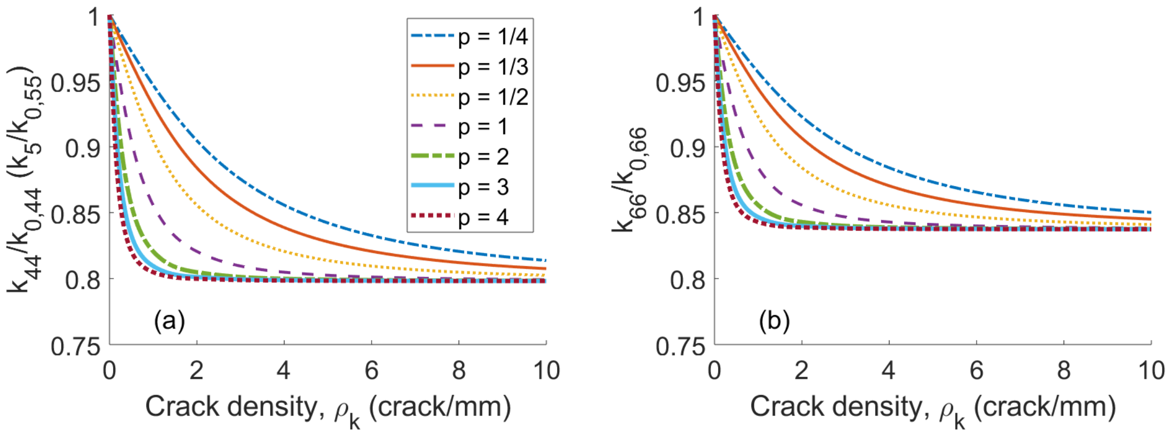

Figure 5: Effect of matrix crack density and ply level scaling on the cross-section (a) bending stiffnesses, k_{44} and k_{55}, and (b) torsional stiffness, k_{66}, of laminated beams made of $[0_p/60_p/0_p/-60_p]_s$ laminates, where p is the number of plies.

Furthermore, the effects of the *sub-laminate scaling* on the stiffness degradation of laminated beams due to matrix cracks are shown in Fig. 6. Contrary to the ply level scaling, the sub-laminate scaling seems not to significantly contribute to the degradation of the beam stiffness. This is because the thickness of all cracked plies of the scaled laminates keep the same than the ones of the initial laminate. Only small variations between the degradation level of the different beams are found, which are due to the orientation of the closest plies to the symmetric plane. For the evaluated cases, the closest plies to the symmetric plane are oriented either at $-\theta$ or 0° depending on how many times the base laminate is multiplied.

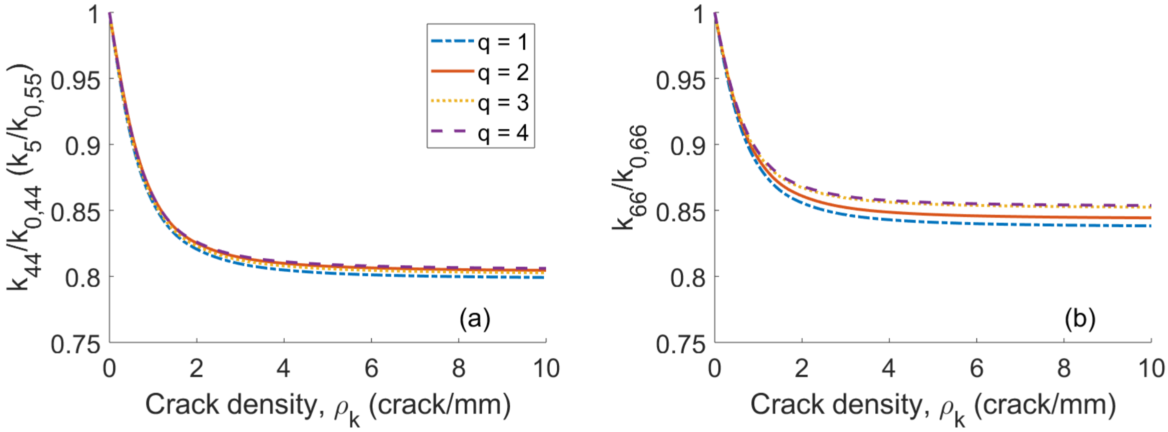

Figure 6: Effect of matrix crack density and sub-laminate scaling on the cross-section (a) bending stiffnesses, k_{44} and k_{55}, and (b) torsional stiffness, k_{66}, of laminated beams made of $[0/60/0/-60]_{s,q}$ where q is the number of sub-laminates.

These results show that, in order to avoid a considerable degradation of the beam properties due to matrix cracks, it is better to use laminates made of thin plies and to use one base layup sequence several times rather than having the same base layup only once but with thicker plies. However, it is worth to note that a greater number of matrix cracks can initiate in laminates with thin plies if compared with laminates with thick plies [25]. This can increase the possibility that other damage mechanisms, such as delaminations [1, 6, 15, 26], develop and start not only contributing to the degradation of the beam properties but also triggering the final failure, see Fig. 1. Hence, a balance between the ply thickness and the development of different damage mechanisms must be considered during the design process of the laminated beams.

3.3 Cross-section geometry effect

So far, it has been assumed that all laminates within the beams are the same and degrade in the same way. This has not only caused that $k_{44}/k_{0,44}$ and $k_{55}/k_{0,55}$ were the same with each other (see Fig. 4-6), but also that the degradation level of the bending and torsion stiffness were the same for all cross-sections regardless of their geometry. That is why it was not specified in sections 3.1 and 3.2 what cross-section geometries were analyzed. Nevertheless, in reality, the laminates within laminated composite beams (especially in wind turbine blades) do not necessarily degrade in the same way. This can be due to possible complex external loads and to possible internal defects induced during the manufacturing process, among other factors. That is why this section analyzes how the cross-section geometry can affect the bending stiffness degradation level of laminated beams with the presence of matrix cracks when their laminates degrade differently between each other. The cross-section geometry effect on the beam torsional stiffness degradation is out the scope of this paper and it is expected to be published in a future scientific paper.

In order to more easily explain the cross-section geometry effect on the beam bending stiffness degradation, the structural response of the thin-walled rectangular box (TWRB) cross-section and the thin-walled semi-rectangular box (TWSRB) cross-section (see Fig. 3) are compared to each other in Fig. 7. For this, the degradation level of their stiffness properties was estimated when only the caps (upper and lower laminates) or only the webs are degraded. The response when all laminates are degraded in the same way is also shown in Fig. 7 in order to have a reference point.

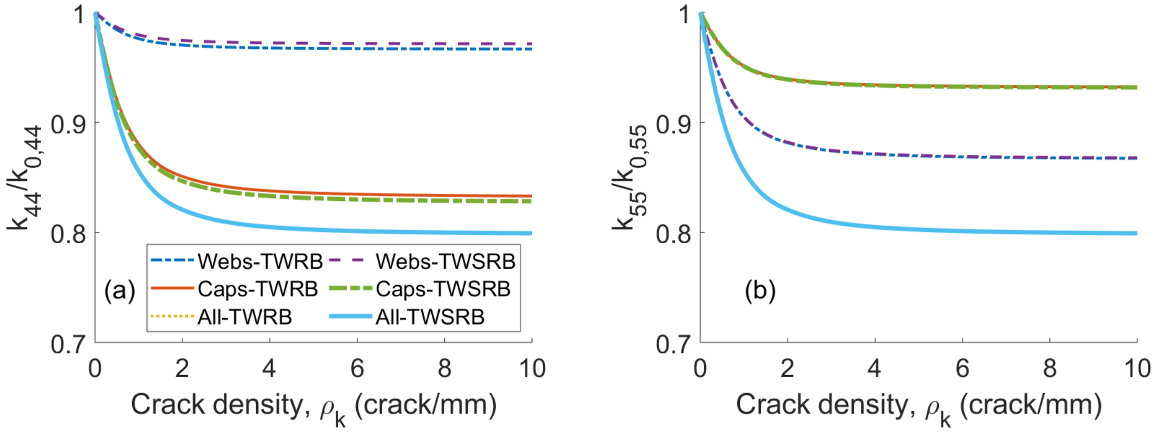

Figure 7: Effect of matrix crack density and cross-section geometry on the (a) bending stiffness, k_{44}, and (b) bending stiffness, k_{55}, of thin-walled rectangular box (TWRB) and thin-walled semi-rectangular box (TWSRB) cross-sections made of $[0/60/0/-60]_s$ laminates.

Fig. 7-a and 7-b together with Table 2 show, for example, how each of the laminates contributes to the degradation of the bending stiffness in a different way depending on their area moment of inertia. The larger the area moment of inertia of the cracked laminate (i.e., the bigger the laminate cross-section area and/or the farther the location of the laminate centroid respect to the centroid of the cross-section), the greater the contribution of this laminate on the bending stiffness degradation of the cross-section. For example, for the two beams, the cracked caps contribute more to the $k_{44}/k_{0,44}$ degradation level than the cracked webs (see Fig. 7-a) because the area moment of inertia of the caps about the *x*-axis, I_{xx}, is larger than I_{xx} of the webs, see Table 2. Whereas, for the two beams as well, the cracked webs have a greater effect on the $k_{55}/k_{0,55}$ degradation level than the cracked caps (see Fig. 7-b) because the area moment of inertia of the webs about the *y*-axis, I_{yy}, is larger than I_{yy} of the caps, see Table 2.

		I_{xx} (m^4)	I_{yy} (m^4)	$A_l(m^2)$
TWRB	Caps	5.5270E-6	4.3948E-6	1.40E-3
	Webs	1.1300E-6	8.6465E-6	8.69E-4
TWSRB	Caps	6.8222E-6	4.4875E-6	1.50E-3
	Webs	1.1302E-6	8.6466E-6	8.69E-4
Airfoil	Caps	6.8224E-6	4.4877E-6	1.50E-3
	Webs	1.1301E-6	8.6464E-6	9.50E-4
	TE	3.4363E-6	1.4118E-4	2.10E-3
	LE	1.7656E-6	2.4228E-5	8.94E-4

Table 2: Area moment of inertia about the *x*-axis, I_{xx}, area moment of inertia about the *y*-axis, I_{yy}, and area, A_l, of different regions within the evaluated cross-sections.

Moreover, Fig. 7-a also shows that the degradation level $k_{44}/k_{0,44}$ of the TWSRB cross-section is slightly greater than the one of the TWRB cross-section. This small difference is due to the fact that the area moment of inertia I_{xx} of the TWSRB caps is slightly larger than the area moment of inertia I_{xx} of the TWRB caps (see Table 2) due to their semi-arc geometry. In addition, note that the small decrease of the degradation level $k_{44}/k_{0,44}$ of the TWSRB cross-section respect to the TWRB cross-section is a compensation for the increase in the contribution of the cracked caps to the total degradation level of the cross-section. Regarding the degradation level $k_{55}/k_{0,55}$ (see Fig. 7-b), no differences between the two cross-sections were found because the area moment of inertia I_{yy} of the caps are similar in both cases as well as $I_{yy}$of the webs, see Table 2.

These results can be used to better understand how the different regions of wind turbine blades (i.e., caps, shear webs, trailing-edge panels (TE), and leading-edge panels (LE)) contribute to the bending stiffness degradation level of these structures when such regions have off-axis cracks. Fig. 8 shows, for example, these effects on the bending stiffnesses of the analyzed airfoil cross-section. Accordantly, the order in which the cracked regions contribute to the bending stiffness degradation level of the cross-section is directly related to their area moment of inertia. This can be verified by comparing the stiffness degradation level of the airfoil cross-section for different cracked regions (see Fig. 8-a and 8-b) and the area moment of inertia of these regions, see Table 2. For the $k_{44}/k_{0,44}$ case, for example, the cracked caps have the greatest contribution to the stiffness degradation (see Fig. 8-a) because their area moment of inertia I_{xx} is the greatest one respect to the other regions, see Table 2. Whereas, for the $k_{55}/k_{0,55}$ case, for example, the cracked TE has the greatest contribution (see Fig. 8-b) because its area moment of inertia I_{yy} is the greatest one compared to the other regions, see Table 2.

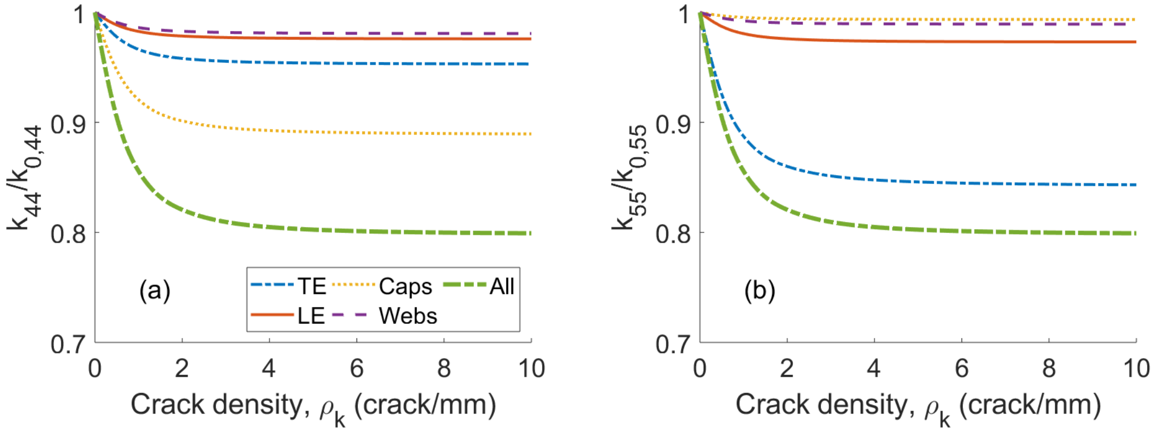

Figure 8: Effect of matrix crack density and cross-section regions on the (a) bending stiffness, k_{44}, and (b) bending stiffness, k_{55}, of an airfoil cross-section made of $[0/60/0/-60]_s$ laminates. Cross-section regions: caps, webs, trailing-edge (TE) and leading-edge (LE).

Finally, it is worth to note that, in reality, a laminate within the cross-sections may not degrade uniformly and, therefore, different crack densities can appear along the length and width of it depending on the internal loading distribution, among other factors. In this sense, each representative volume RVE within the laminate (containing a crack spacing l_k, see Fig. 2) is expected to have an independent effect on the cross-section degradation level. In addition, the use of sandwich panels instead of pure multidirectional laminates (as can occur in the webs, TE and LE of wind turbine blades) may also have a different effect on the beam degradation level. Further studies in these directions may be needed to better understand the structural response of damaged wind turbine blades.

4 CONCLUSIONS

A comprehensive characterization of the effects of matrix cracks on the structural response of laminated composite beams is presented. In this study, the orientation of the cracked plies within the beam laminates, the size of these laminates and the cross-section geometry were considered. It was found that there is a high effect on the degradation level of the beam stiffness properties from the orientation and thickness of the cracked plies. For example, it was found that in order to avoid a considerable degradation of the beam properties due to matrix cracks, it is preferable to use laminates with thin plies and one base layup sequence repeated several times rather than having laminates with the same base layup only once but with thicker plies. Moreover, it was found that the larger the area moment of inertia of a cracked laminate within the cross-section, the greater its contribution on the bending stiffness degradation. All these results can be used to get a better understanding of damaged wind turbine blades, whose geometry and material distribution are complex and vary considerably within the cross-section and along the span-wise direction.

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